Some Formulae Used in the Analysis of Algorithms

The Master Theorem

Suppose

- $r(n) = Cn^d + Ar(n/b)$ where A, B, and C are positive constants
- $r(0) = B$

Then

- If $A < b^d$ then $r(n)$ has $O(n^d)$, $\Theta(n^d)$, $\Omega(n^d)$
- If $A = b^d$ then $r(n)$ has $O(n^d \log n)$, $\Theta(n^d \log n)$, $\Omega(n^d \log n)$
- If $A > b^d$ then $r(n)$ has $O(n^{log_b A})$, $\Theta(n^{log_b A})$, $\Omega(n^{log_b A})$

Polynomial Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} i^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=1}^{n} i^5 = \frac{n^2(2n^2+2n-1)(n+1)^2}{12}$$

Geometric Series

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \quad \text{for} \quad a \neq 1$$

Logarithmic Series

$$\sum_{i=1}^{n} i^k \ \log \ i \ \text{ has } \ \Theta(n^{k+1} \ \log \ n) \ \text{ for } \ k = 0, 1, 2, ...$$