Some Formulae Used in the Analysis of Algorithms

The Master Theorem

Suppose
- \( r(n) = Cn^d + Ar(n/b) \) where \( A, B, \) and \( C \) are positive constants
- \( r(0) = B \)

Then
- If \( A < b^d \) then \( r(n) \) has \( O(n^d), \ \Theta(n^d), \ \Omega(n^d) \)
- If \( A = b^d \) then \( r(n) \) has \( O(n^d \log n), \ \Theta(n^d \log n), \ \Omega(n^d \log n) \)
- If \( A > b^d \) then \( r(n) \) has \( O(n^{\log_b A}), \ \Theta(n^{\log_b A}), \ \Omega(n^{\log_b A}) \)

Polynomial Series

\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\]
\[
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}
\]
\[
\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}
\]
\[
\sum_{i=1}^{n} i^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}
\]
\[
\sum_{i=1}^{n} i^5 = \frac{n^2(2n^2+2n-1)(n+1)^2}{12}
\]

Geometric Series

\[
\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1} \quad \text{for} \quad a \neq 1
\]

Logarithmic Series

\[
\sum_{i=1}^{n} i^k \log i \text{ has } \Theta(n^{k+1} \log n) \quad \text{for} \quad k = 0, 1, 2, \ldots
\]